Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

---

1 What is the domain and range of $y = 2\sin^{-1}(3x)$?

A. Domain: $-3 \le x \le 3$, range: $-\frac{\pi}{4} \le y \le \frac{\pi}{4}$.

B. Domain: $-\frac{1}{3} \le x \le \frac{1}{3}$, range: $-\frac{\pi}{4} \le y \le \frac{\pi}{4}$.

C. Domain: $-3 \le x \le 3$, range: $-\pi \le y \le \pi$.

D. Domain: $-\frac{1}{3} \le x \le \frac{1}{3}$, range: $-\pi \le y \le \pi$.

---

2 Which of the following is equivalent to $\frac{d}{dx}\left[\arccos(2x)\right]$?

A. $-\dfrac{1}{2\sqrt{1-4x^2}}$

B. $-\dfrac{2}{\sqrt{1-4x^2}}$

C. $\dfrac{2}{\sqrt{4x^2-1}}$

D. $\dfrac{1}{2\sqrt{1-4x^2}}$

3 If $t = \tan\frac{\theta}{2}$, which of the following expressions represents $\sec\theta + \tan\theta$?

A. $\frac{1+t}{1-t}$

B. $\frac{1-t}{1+t}$

C. $\left(\frac{1+t}{1-t}\right)^2$

D. $\left(\frac{1-t}{1+t}\right)^2$

---

4 What is the value of $\sin 2A$ given $\tan A = -\frac{5}{12}$ and $-\frac{\pi}{2} < A < 0$?

A. $\frac{60}{169}$

B. $-\frac{60}{169}$

C. $\frac{120}{169}$

D. $-\frac{120}{169}$

---

5 Which of the following is a solution to the differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2$ with initial condition $y(0) = 0$?

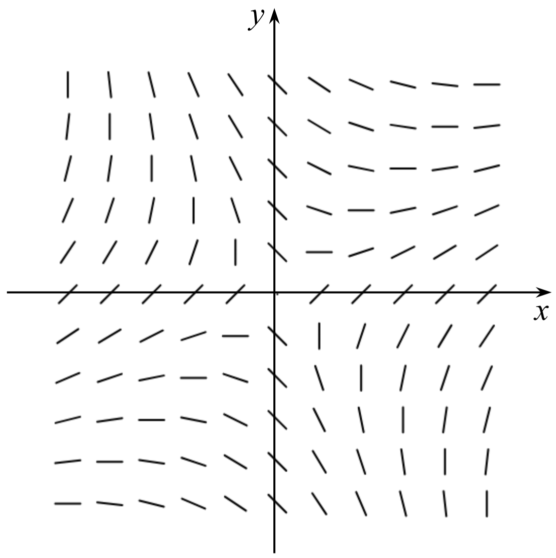
A. $y = \tan\left(x + \frac{x^2}{2}\right)$

B. $y = \tan\left(\ln|1+x|\right)$

C. $y = e^{x+\frac{x^2}{2}} - 1$

D. $y = x$

6 Consider the following slope field.



Which of the following differential equations best describes the given slope field?

A. $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$

B. $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$

C. $\dfrac{dy}{dx} = (x+y)(x-y)$

D. $\dfrac{dy}{dx} = \dfrac{1}{(x+y)(x-y)}$

---

7 Which of the following statements is true if m and n are integers and $m > n > 0$?

A. $\int_0^{\pi} \sin^2 mx\,dx > \int_0^{\pi} \cos^2 nx\,dx$

B. $\int_0^{\pi} \cos^2 mx\,dx > \int_0^{\pi} \sin^2 nx\,dx$

C. $m\int_0^{\pi} \sin^2 x\,dx > n\int_0^{\pi} \cos^2 x\,dx$

D. $m\int_0^{\pi} \sin^2 x\,dx < n\int_0^{\pi} \cos^2 x\,dx$

8 Consider unit vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, given that $\underset{\sim}{a}+\underset{\sim}{b}$ is also a unit vector, what is the value of $|\underset{\sim}{a}-\underset{\sim}{b}|$?

A. 1

B. $\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. $\sqrt{2}$

---

9 Consider a random variable *X* which follows a binomial distribution with parameters *n* and *p*, where *n* is the number of independent trials and *p* is the probability of success with $0<p<1$.

Given $\frac{P(X=r)}{P(X=n-r)}$ is independent of *n* and *r*. What is the value of *p*?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{3}{4}$

---

10 Consider two non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, which of the following relationships will guarantee the result $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b}=\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}$?

A. $\underset{\sim}{a}=-\underset{\sim}{b}$

B. $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$

C. $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$

D. $|\underset{\sim}{a}|=|\underset{\sim}{b}|$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 section of the writing booklet.

(a) Find $\int \frac{dx}{16+25x^2}$. **2**

(b) Find the exact volume of the solid of revolution formed when the region enclosed by the curve $y = 3x - x^2$ and the x-axis is rotated about the x-axis. **3**

(c) Calculate the angle between $\underset{\sim}{a} = 6\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + 2\underset{\sim}{j}$ correct to the nearest degree. **2**

(d) (i) By considering $\tan 3A$ as $\tan(2A + A)$, show that **2**

$$\tan A \tan 2A \tan 3A = \tan 3A - \tan 2A - \tan A .$$

(ii) Hence solve $\tan 3A - \tan 2A - \tan A = 0$, where $0 \le A \le \pi$. **2**

(e) (i) Express $3\sin x - 5\cos x$ in the form $A\sin(x - \alpha)$ where $A > 0$ and $0° \le \alpha \le 90°$. **2**

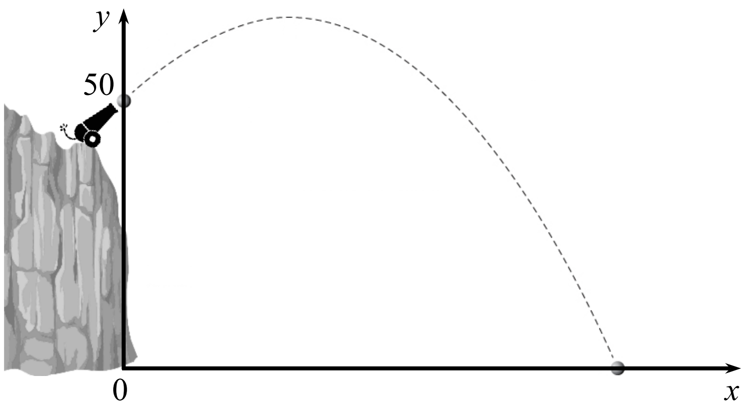
(ii) Hence solve $3\sin x - 5\cos x = -1$ correct to the nearest minute for $0° \le x \le 360°$. **2**

End of Question 11

Question 12 (15 marks) Use the Question 12 section of the writing booklet.

(a) A cannonball is fired from a cannon sitting on the top of a cliff into the sea. Initially, the cannonball is 50 metres above the foot of the cliff and its velocity vector at that time is $30\sqrt{3}\underset{\sim}{i} + 15\underset{\sim}{j}$, measured in metres per second.

The acceleration due to gravity is a constant 9.8 m/s^2 for the duration of the cannonball's flight and air resistance is negligible.



Use the foot of the cliff as the origin as shown in the diagram above. You are also given that the velocity vector of the cannonball t seconds after it was fired is:

$$\underset{\sim}{v}(t) = \left(30\sqrt{3}\right)\underset{\sim}{i} + (15 - 9.8t)\underset{\sim}{j}. \qquad \text{(Do NOT prove this.)}$$

(i) Calculate the initial speed of the cannonball correct to the nearest metre per second. **1**

(ii) Find the size of the angle at which the cannonball was fired correct to the nearest degree. **1**

(iii) Write down the position vector, $\underset{\sim}{r}(t)$, of the cannonball. **2**

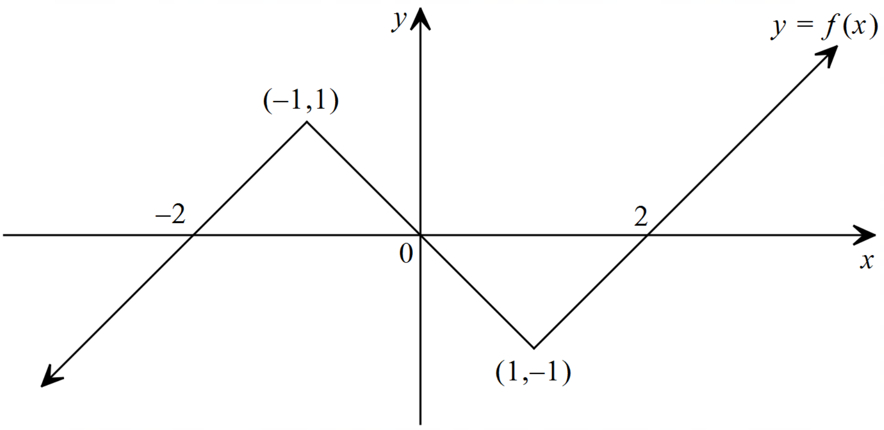
(iv) Determine the greatest height above sea level achieved by the cannonball. Express your answer correct to 2 decimal places. **2**

(v) How long will it take for the cannonball to land on the surface of the sea? Express your answer correct to 2 decimal places. **1**

Question 12 continues on the next page

Question 12 (continued)

(b) Consider the graph of $y = f(x)$ below.



On separate number planes, graph of each of the following:

(i) $y = \dfrac{1}{f(x)}$ **2**

(ii) $y = f(|x| - 2)$ **2**

(c) In how many ways can the letters of the word ***DIFFERENCE*** be arranged if:

(i) there are no restrictions? **2**

(ii) all three ***E***s are next to each other with no letters separating them, and this string of three ***E***s are somewhere between the two ***F***s? **2**

End of Question 12

Question 13 (15 marks) Use the Question 13 section of the writing booklet.

(a) Prove by mathematical induction that, for all positive integers n, **3**

$$2 \times 6 \times 10 \times \cdots \times (4n-2) = \frac{(2n)!}{n!}.$$

(b) Consider the polynomial $f(x) = x^3 + (a+2)x^2 - 2x + b$ where a and b are non-zero integers.

It is given that $(x-2)$ and $(x+a)$ are both factors of $f(x)$ with $a > 0$.

(i) Justify why $4a + b = -12$. **1**

(ii) Show also that $2a^2 + 2a + b = 0$. **1**

(iii) Hence evaluate the integers a and b. **2**

(iv) Fully factorise $f(x)$ and graph $y = f(x)$, showing all intercepts with the coordinate axes. You do NOT need to show the coordinates of turning points. **2**

(c) Consider the differential equation $y'' - 2y' - 3y = 0$.

(i) For any constant k, show that if $y = e^{kx}$ is a solution to this differential equation, then $k^2 - 2k - 3 = 0$. **2**

(ii) Hence find the solutions to the differential equation that are of the form $y = e^{kx}$ where k is a constant. **2**

(iii) Suppose k_1 and k_2 are the values of k for which $y = e^{kx}$ is a solution to the differential equation. Show that $y = Ae^{k_1 x} + Be^{k_2 x}$, where A and B are constants, is also a solution to the same differential equation. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 section of the writing booklet.

(a) Given two non-zero vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$, show that $\dfrac{\left|\underset{\sim}{p}+\underset{\sim}{q}\right|^2+\left|\underset{\sim}{p}-\underset{\sim}{q}\right|^2}{\left|\underset{\sim}{p}\right|^2+\left|\underset{\sim}{q}\right|^2}=2$. **2**

(b) (i) Use the substitution $u=9-\sqrt{h}$ to show $\displaystyle\int\frac{dh}{9-\sqrt{h}}=-18\ln\left|9-\sqrt{h}\right|-2\sqrt{h}+c$ **2**

where c is a constant.

A team of scientists is studying a species of growing trees. The rate of change in height of a tree of this species is modelled by the differential equation

$$\frac{dh}{dt}=\frac{t^{0.5}\left(9-\sqrt{h}\right)}{18}$$

where h is the height of the tree in metres and t is the time, measured in years, after the tree was planted.

(ii) According to the model, what is the range in heights of trees of this species? **1**

(iii) One of these trees is one metre high when it was first planted. According to this model, calculate the time this tree would take to reach a height of 16 metres, giving your answer correct to 3 significant figures. **3**

(c) A particular variety of rose bushes in a garden have been studied over a long period of time. Each rose bush grows roses, all in one of two colours, red or pink. It has been determined that each rose bush of this particular variety has a 15% chance of growing pink roses.

(i) A selection of 10 such rose bushes are bought, find the probability that exactly 3 of these bushes will grow pink roses. **1**

(ii) Calculate the least number of rose bushes that need to be bought so that the the probability of at least one rose bush growing pink flowers exceeds 0.95. **2**

(iii) A new customer has bought 125 of this variety of rose bushes, using a normal approximation, determine the probability that the number of rose bushes that will grow pink flowers will be at least 20 but no more than 30. You may wish to use the z-table provided to answer this question. **4**

End of Paper

YEAR 12 TRIAL EXAMINATION 2024
MATHEMATICS EXTENSION 1
MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	B
3	A
4	D
5	A

Question	Answer
6	B
7	C
8	B
9	A
10	C

Questions 1 – 10

Sample solution

1. Domain:

$$-1 \le 3x \le 1$$
$$-\frac{1}{3} \le x \le \frac{1}{3}$$

Range:

$$-\frac{\pi}{2} \le \sin^{-1}(3x) \le \frac{\pi}{2}$$
$$-\pi \le 2\sin^{-1}(3x) \le \pi$$

2.

$$\frac{d}{dx}\left[\arccos f(x)\right] = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$$
$$\frac{d}{dx}\left[\arccos(2x)\right] = -\frac{2}{\sqrt{1-(2x)^2}}$$
$$= -\frac{2}{\sqrt{1-4x^2}}$$

3.

$$\sec\theta + \tan\theta = \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$$
$$= \frac{1+2t+t^2}{1-t^2}$$
$$= \frac{(1+t)^2}{(1-t)(1+t)}$$
$$= \frac{1+t}{1-t}$$

4. If $\tan A = -\frac{5}{12}$ and A is in the fourth quadrant $\left(-\frac{\pi}{2} < A < 0\right)$, then $\sin A = -\frac{5}{13}$ and $\cos A = \frac{12}{13}$.

$$\sin 2A = 2\sin A\cos A$$
$$= 2\times\left(-\frac{5}{13}\right)\times\frac{12}{13}$$
$$= -\frac{120}{169}$$

5.

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$
$$= 1 + x + y^2(1+x)$$
$$= (1+x)(1+y^2)$$
$$\int \frac{dy}{1+y^2} = \int 1 + x\,dx$$
$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

When $x = 0, y = 0$:

$$\tan^{-1} 0 = 0 + \frac{0^2}{2} + c$$
$$0 = 0 + c$$
$$c = 0$$

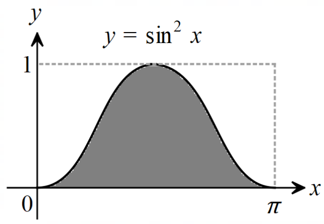
$$\tan^{-1} y = x + \frac{x^2}{2}$$
$$y = \tan\left(x + \frac{x^2}{2}\right)$$

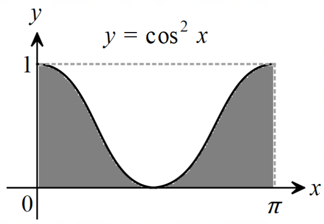
6. From the slope field, $\frac{dy}{dx} = 0$ when $y = x$ and $\frac{dy}{dx}$ is undefined when $y = -x$, so the differential equation that best describes the given slope field is $\frac{dy}{dx} = \frac{x-y}{x+y}$.

Sample solution

7.

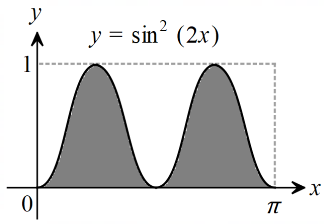
To the right are the curves of $y = \sin^2 x$ and $y = \cos^2 x$, noting both have a period of π units. The shaded area in each graph is exactly half that of the bounding rectangle drawn using rotational symmetry or by dissecting and rearranging, and is therefore each equal to $\frac{\pi}{2}$ square units.

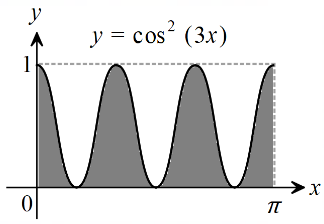




Note: $\sin^2 x = \tfrac{1}{2}(1-\cos 2x)$ *and* $\cos^2 x = \tfrac{1}{2}(1+\cos 2x)$

Compressing either graph by an integer factor of *m* (or *n*) will simply introduce more complete cycles of the graph, but the area under each graph from $x = 0$ to $x = \pi$ will remain to be half of the bounding rectangle, i.e. $\frac{\pi}{2}$ square units.





$$\therefore \int_0^\pi \sin^2(mx) = \int_0^\pi \cos^2(mx) = \int_0^\pi \sin^2(nx) = \int_0^\pi \cos^2(nx) = \frac{\pi}{2}$$

An algebraic approach confirms, for any integer *k*:

$$\begin{aligned}
&\int_0^\pi \sin^2 kx\,dx \\
&= \frac{1}{2}\int_0^\pi 1-\cos 2kx\,dx \\
&= \frac{1}{2}\left[x+\frac{1}{2k}\sin 2kx\right]_0^\pi \\
&= \frac{1}{2}\left(\pi+\frac{1}{2}\sin 2k\pi - 0\right) \\
&= \frac{\pi}{2}
\end{aligned}$$

$$\begin{aligned}
&\int_0^\pi \cos^2 kx\,dx \\
&= \frac{1}{2}\int_0^\pi 1+\cos 2kx\,dx \\
&= \frac{1}{2}\left[x-\frac{1}{2k}\sin 2kx\right]_0^\pi \\
&= \frac{1}{2}\left(\pi-\frac{1}{2}\sin 2k\pi - 0\right) \\
&= \frac{\pi}{2}
\end{aligned}$$

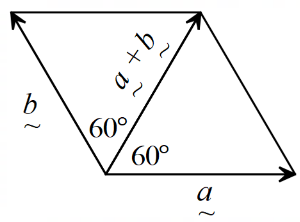
With this equality in mind,

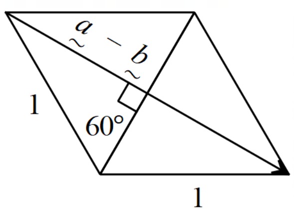
$$m > n$$

$$\therefore m\int_0^\pi \sin^2 x > n\int_0^\pi \cos^2 x$$

8.

The only configuration for $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{a}+\underset{\sim}{b}$ to all be unit vectors is:





($\underset{\sim}{a}$ and $\underset{\sim}{b}$ interchangeable)

In either case, the long diagonal of this rhombus is $\underset{\sim}{a}-\underset{\sim}{b}$, of which the length is $\sqrt{3}$ using right-angled trigonometry.

Questions 1 – 10 (continued)

Sample solution	
9.	Let $q = 1 - p$ be the probability of failure. $\dfrac{P(X=r)}{P(X=n-r)} = \dfrac{{}^{n}C_{r}\,p^{r}q^{n-r}}{{}^{n}C_{n-r}\,p^{n-r}q^{r}} = \dfrac{q^{n-2r}}{p^{n-2r}} = \left(\dfrac{q}{p}\right)^{n-2r}$ For this quantity to be independent of *n* and *r*, the value of $\left(\dfrac{q}{p}\right)^{n-2r}$ should be constant regardless of the choice of *n* and *r*. This occurs if either $\dfrac{q}{p} = 0$ or $\dfrac{q}{p} = 1$. If $\dfrac{q}{p} = 0$, then $q = 0$ and $p = 1$, but $0 < p < 1$, so $\dfrac{q}{p} \neq 0$. Therefore $\dfrac{q}{p} = 1$, implying $p = q$. Since $p + q = 1$, then $p = q = \dfrac{1}{2}$.
10.	By the geometry of vector projections, if $\underset{\sim}{a} = -\underset{\sim}{b}$, then $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \underset{\sim}{b}$ and $\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \underset{\sim}{a}$. Similarly, if $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$, then $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \underset{\sim}{b}$ and $\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \underset{\sim}{a}$. The projection of a vector onto another vector that is parallel to it will result in the original vector. If $\lvert \underset{\sim}{a} \rvert = \lvert \underset{\sim}{b} \rvert$, there is no guarantee that $\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \text{proj}_{\underset{\sim}{a}}\underset{\sim}{b}$. One instance for when $\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \text{proj}_{\underset{\sim}{a}}\underset{\sim}{b}$ is the trivial case where $\underset{\sim}{a} = \underset{\sim}{b}$. Another instance is if $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$. In this instance, $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \underset{\sim}{0}$ and $\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \underset{\sim}{0}$, making $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}$. In fact, whenever $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$, $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \underset{\sim}{0}$ and $\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \underset{\sim}{0}$, the lengths of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are irrelevant. Algebraically, $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}$ $\left(\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{\underset{\sim}{a}\cdot\underset{\sim}{a}}\right)\underset{\sim}{a} = \left(\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\right)\underset{\sim}{b}$ Trivial if $\underset{\sim}{a} = \underset{\sim}{b}$. Otherwise, suppose $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$, then $\underset{\sim}{a} = k\underset{\sim}{b}$ for some constant *k*. $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}$ $\left(\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{\underset{\sim}{a}\cdot\underset{\sim}{a}}\right)\underset{\sim}{a} = \left(\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\right)\underset{\sim}{b}$ $\left(\dfrac{\underset{\sim}{b}\cdot k\underset{\sim}{b}}{k\underset{\sim}{b}\cdot k\underset{\sim}{b}}\right)k\underset{\sim}{b} = \left(\dfrac{k\underset{\sim}{b}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\right)\underset{\sim}{b}$ $\dfrac{1}{k}\left(\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\right)k\underset{\sim}{b} = k\left(\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\right)\underset{\sim}{b}$ $1 = k$ That is, $\underset{\sim}{a} = \underset{\sim}{b}$, once again resulting in the trivial case. If $\underset{\sim}{a}$ is not parallel to $\underset{\sim}{b}$, then $\left(\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{\underset{\sim}{a}\cdot\underset{\sim}{a}}\right)\underset{\sim}{a} = \left(\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\right)\underset{\sim}{b}$ only if $\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{\underset{\sim}{a}\cdot\underset{\sim}{a}} = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}} = 0$, i.e. $\underset{\sim}{a}\cdot\underset{\sim}{b} = 0$, meaning that $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$.

Section II

Question 11

	Sample solution	Suggested marking criteria
(a)	$\int \frac{dx}{16+25x^2} = \frac{1}{16}\int \frac{dx}{1+\frac{25x^2}{16}}$ $= \frac{1}{20}\int \frac{\frac{5}{4}dx}{1+\left(\frac{5x}{4}\right)^2}$ $= \frac{1}{20}\tan^{-1}\left(\frac{5x}{4}\right)+c$	• 2 – correct solution • 1 – recognises the primitive is an inverse tangent function
(b)	$V = \pi\int_0^3 \left(3x-x^2\right)^2 dx$ $= \pi\int_0^3 9x^2-6x^3+x^4 dx$ $= \pi\left[3x^3-\frac{3x^4}{2}+\frac{x^5}{5}\right]_0^3$ $= \pi\left(3\times 3^3-\frac{3\times 3^4}{2}+\frac{3^5}{5}-0\right)$ $= \pi\left(81+\frac{243}{2}+\frac{243}{5}\right)$ $= \frac{81\pi}{10}$ cubic units	• 3 – correct solution • 2 – correctly integrates an appropriate integrand • 1 – correct integral for the volume of the solid of revolution
(c)	$\cos\theta = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert}$ $= \frac{\left(6\underset{\sim}{i}-\underset{\sim}{j}\right)\cdot\left(-3\underset{\sim}{i}+2\underset{\sim}{j}\right)}{\lvert 6\underset{\sim}{i}-\underset{\sim}{j}\rvert\lvert -3\underset{\sim}{i}+2\underset{\sim}{j}\rvert}$ $= \frac{6\times(-3)+(-1)\times 2}{\sqrt{6^2+(-1)^2}\times\sqrt{(-3)^2+2^2}}$ $= \frac{-20}{\sqrt{37}\times\sqrt{13}}$ $\theta = \cos^{-1}\left(\frac{-20}{\sqrt{481}}\right)$ $= 156^\circ$ (nearest degree)	• 2 – correct solution • 1 – correctly evaluates $\lvert\underset{\sim}{a}\rvert$, $\lvert\underset{\sim}{b}\rvert$ or $\underset{\sim}{a}\cdot\underset{\sim}{b}$

Question 11 (continued)

		Sample solution	Suggested marking criteria
(d)	(i)	$\tan 3A = \tan(2A + A)$ $= \dfrac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$ $\tan 3A - \tan 3A \tan 2A \tan A = \tan 2A + \tan A$ $\tan 3A - \tan 2A - \tan A = \tan A \tan 2A \tan 3A$	• 2 – correct solution • 1 – correctly applies the tangent of a compound angle formula
	(ii)	$\tan 3A - \tan 2A - \tan A = 0$ $\tan A \tan 2A \tan 3A = 0$ $\tan A = 0$ $\tan 2A = 0$ $\tan 3A = 0$ $A = 0, \pi$ $2A = 0, \pi, 2\pi$ $3A = 0, \pi, 2\pi, 3\pi$ $A = 0, \frac{\pi}{2}, \pi$ $A = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$ $A \neq \frac{\pi}{2}$ as this is not well-defined for the original problem with a $\tan\phi$ term. $\therefore A = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$	• 2 – correct solution • 1 – correctly finds some valid values of A
(e)	(i)	$3\sin x - 5\cos x \equiv A\sin(x - \alpha)$ $= A\sin x\cos\alpha - A\cos x\sin\alpha$ Equating coefficients of the $\sin x$ and $\cos x$ terms gives $A\sin\alpha = 5$ and $A\cos\alpha = 3$. $(A\sin\alpha)^2 + (A\cos\alpha)^2 = 5^2 + 3^2$ $A^2\sin^2\alpha + A^2\cos^2\alpha = 25 + 9$ $A^2(\sin^2\alpha + \cos^2\alpha) = 34$ $A^2 = 34$ $A = \sqrt{34}$ $(\text{since } A > 0)$ $\dfrac{A\sin\alpha}{A\cos\alpha} = \dfrac{5}{3}$ $\tan\alpha = \dfrac{5}{3}$ $\alpha = \tan^{-1}\left(\dfrac{5}{3}\right)$ $= 59°2'$ (nearest minute) $\therefore 3\sin x - 5\cos x = \sqrt{34}\sin(x - 59°2')$ (nearest minute)	• 2 – correct solution • 1 – correct value of A or α
	(ii)	$3\sin x - 5\cos x = -1$ $\sqrt{34}\sin(x - 59°2') = -1$ $\sin(x - 59°2') = \dfrac{-1}{\sqrt{34}}$ $x - 59°2' = -9°52'$, $189°52'$ $x = 49°10'$, $248°54'$ (nearest minute)	• 2 – correct solution • 1 – correctly find a valid value of x

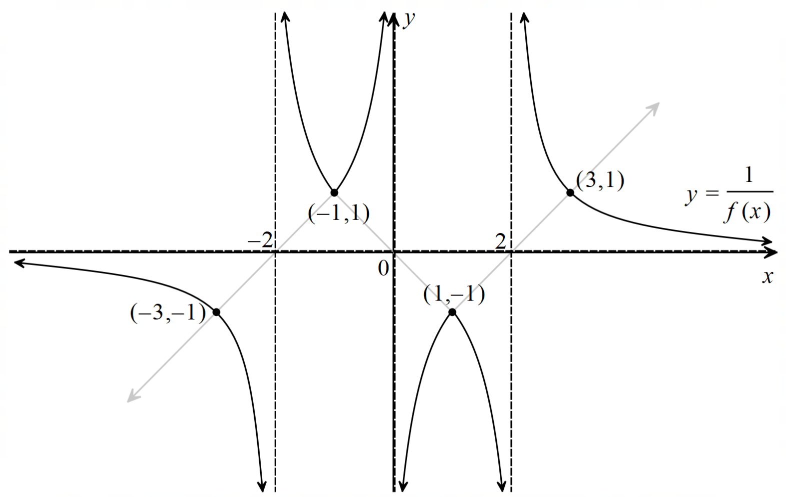
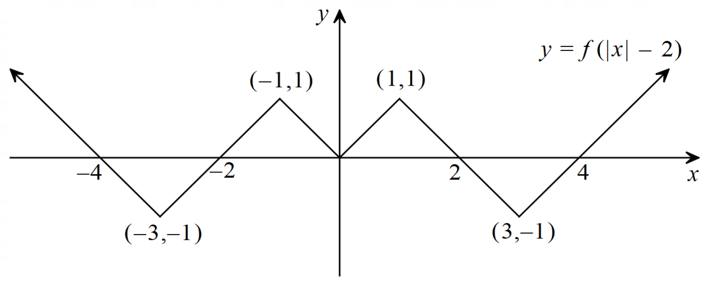
Question 12

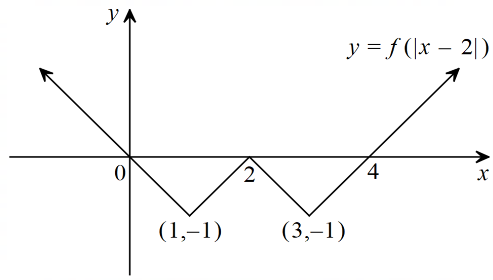
<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td rowspan="5">(a)</td><td>(i)
Initial speed $= \sqrt{\left(30\sqrt{3}\right)^2 + 15^2}$
$= \sqrt{2700 + 225}$
$= \sqrt{2925}$
$= 54$ m/s (nearest m/s)</td><td>• 1 – correct solution</td></tr>
<tr><td>(ii)
angle of projection $= \tan^{-1}\left(\dfrac{15}{30\sqrt{3}}\right)$
$= 16°$ (nearest degree)</td><td>• 2 – correct solution
• 1 – correct solution</td></tr>
<tr><td>(iii)
$\underset{\sim}{v}(t) = \left(30\sqrt{3}\right)\underset{\sim}{i} + (15 - 9.8t)\underset{\sim}{j}$
$\underset{\sim}{r}(t) = \left(30\sqrt{3}t\right)\underset{\sim}{i} + \left(15t - 4.9t^2\right)\underset{\sim}{j} + \underset{\sim}{c}$

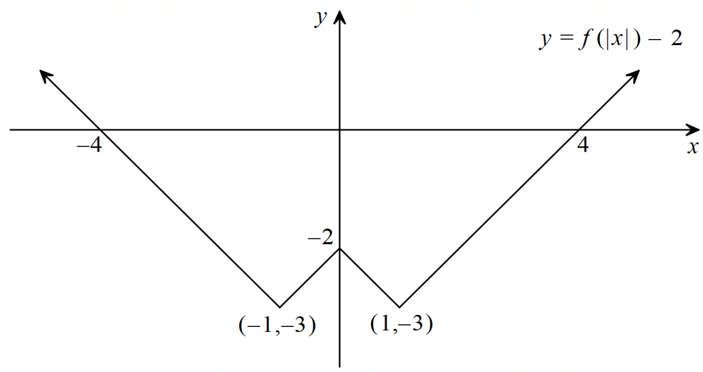
$\underset{\sim}{r}(0) = 50\underset{\sim}{j}$, therefore $\underset{\sim}{c} = 50\underset{\sim}{j}$:
$\underset{\sim}{r}(t) = \left(30\sqrt{3}t\right)\underset{\sim}{i} + \left(50 + 15t - 4.9t^2\right)\underset{\sim}{j}$</td><td>• 2 – correct solution
• 1 – correctly finds either the horizontal or the vertical component of the position vector</td></tr>
<tr><td>(iv)
Greatest height occurs when $\dot{y}(t) = 0$:
$15 - 9.8t = 0$
$9.8t = 15$
$t = 1.53$ s (2 d.p.)

$y(1.53) = 50 + 15 \times 1.53 - 4.9 \times 1.53^2$
$= 61.48$ m (2 d.p.)</td><td>• 2 – correct solution
• 1 – correctly finds the time when greatest height is achieved</td></tr>
<tr><td>(v)
$y = 0$ when $50 + 15t - 4.9t^2 = 0$:
$t = \dfrac{-15 \pm \sqrt{15^2 - 4 \times (-4.9) \times 50}}{2 \times (-4.9)}$
$= 5.07$ s (2 d.p.)</td><td>• 1 – correct solution</td></tr>
</table>

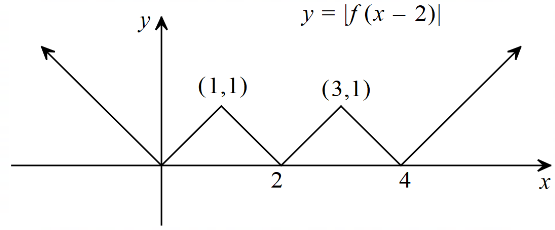
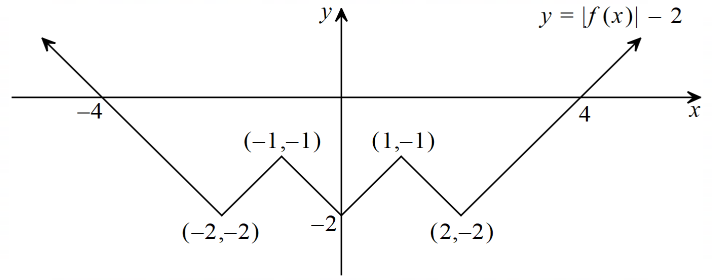
Question 12 (continued)

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(b)</td><td>(i)

</td><td>• 2 – correct solution
• 1 – correctly identifies the vertical asymptotes and $(x,\pm1)$ on the reciprocal graph</td></tr>
<tr><td></td><td>(ii) $y=f(x)\xrightarrow[\text{2 units right}]{\text{translate}}y=f(x-2)\xrightarrow[\text{the } x\text{-axis onto the left}]{\text{reflect what's right of}}y=f(|x|-2)$


The order of the transformations is important, careful not to confuse the solution with:
$y=f(x)\xrightarrow[\text{the } x\text{-axis onto the left}]{\text{reflect what's right of}}y=f(|x|)\xrightarrow[\text{2 units right}]{\text{translate}}y=f(|x-2|)$


$y=f(x)\xrightarrow[\text{the } x\text{-axis onto the left}]{\text{reflect what's right of}}y=f(|x|)\xrightarrow[\text{2 units down}]{\text{translate}}y=f(|x|)-2$

</td><td>• 2 – correct solution
• 1 – recognises a horizonal translation of 2 units to the right
– recognises a reflection in the y-axis</td></tr>
</table>

Question 12 (continued)

		Sample solution	Suggested marking criteria
(c)	(ii)	*(continued)* ***The order of the transformations is important, careful not to confuse the solution with:*** $y=f(x)\xrightarrow[\text{the } y\text{-axis above}]{\text{reflect what's below}} y=\lvert f(x)\rvert \xrightarrow[\text{2 units right}]{\text{translate}} y=\lvert f(x-2)\rvert$  $y=f(x)\xrightarrow[\text{the } y\text{-axis above}]{\text{reflect what's below}} y=\lvert f(x)\rvert \xrightarrow[\text{2 units down}]{\text{translate}} y=\lvert f(x)\rvert-2$ 	
(d)	(i)	Without restrictions, ***DIFFERENCE*** is a 10-letter word, with 2 letter ***F***s and 3 letter ***E***s, therefore, there are $\frac{10!}{2!\times 3!}=302400$ arrangements.	• 2 – correct solution • 1 – arranges the letters of the word without considering all of the repeated letters
	(ii)	Group the ***E***s together, arrange ***D, I, F, F, R, N, C, (EEE)*** in a row in $\frac{8!}{2!}=20160$ ways. Of these arrangements, one third of them will have both ***F***s before the ***(EEE)***, one third of them will have both ***F***s after the ***(EEE)*** and the other third will have the ***(EEE)*** somewhere in between the two ***F***s. Therefore, the total number of arrangements satisfying the given criteria is $\frac{1}{3}\times 20160=6720$. Alternatively, start with ***F (EEE) F*** and the remaining letters (***D***, ***I***, ***R***, ***N***, ***C***) can be inserted in $4\times5\times6\times7\times8=6720$ ways. (This first of the remaining letters can go in front of the first ***F***, in between the first ***F*** and the ***EEE***, in between the ***EEE*** and the second ***F***, or after the last ***F***. Once the first of the remaining letters has been inserted, the second of the remaining letters now has 5 "spaces" in between already placed letters to be inserted into, etc.)	• 2 – correct solution • 1 – finds the total number of ways of arranging the letters grouping the three ***E***s into a string – lists some valid cases that satisfy the given criteria and attempts to evaluate the total number of such arrangements

Question 12 (continued)

Sample solution			Suggested marking criteria
(c)	(ii)	*(continued)*	

Treating ***(EEE)*** as one item, there are 6 positions for the ***(EEE)***, place the ***F***s (one on each side of the ***(EEE)***) and arrange the remaining letters:

_ ***(EEE)*** _ _ _ _ _ _ $\quad \underbrace{1}_{\substack{\text{F left}\\\text{of (EEE)}}} \times \underbrace{6}_{\substack{\text{F right}\\\text{of (EEE)}}} \times \underbrace{5!}_{\substack{\text{remaining}\\\text{letters}}} = 6\times 5!$

_ _ ***(EEE)*** _ _ _ _ _ $\quad \underbrace{2}_{\substack{\text{F left}\\\text{of (EEE)}}} \times \underbrace{5}_{\substack{\text{F right}\\\text{of (EEE)}}} \times \underbrace{5!}_{\substack{\text{remaining}\\\text{letters}}} = 10\times 5!$

_ _ _ ***(EEE)*** _ _ _ _ $\quad \underbrace{3}_{\substack{\text{F left}\\\text{of (EEE)}}} \times \underbrace{4}_{\substack{\text{F right}\\\text{of (EEE)}}} \times \underbrace{5!}_{\substack{\text{remaining}\\\text{letters}}} = 12\times 5!$

_ _ _ _ ***(EEE)*** _ _ _ $\quad 4\times 3\times 5! = 12\times 5!$

_ _ _ _ _ ***(EEE)*** _ _ $\quad 5\times 2\times 5! = 10\times 5!$

_ _ _ _ _ _ ***(EEE)*** _ $\quad 6\times 1\times 5! = 6\times 5!$

$$\begin{aligned}\text{Total number of ways} &= (6+10+12+12+10+6)\times 5!\\ &= 6720\end{aligned}$$

There are 6 ways to leave 3 spaces between the ***F***s, the ***(EEE)*** can be placed between the ***F***s in 1 way each:

F _ _ _ ***F*** _ _ _ _ _
_ ***F*** _ _ _ ***F*** _ _ _ _
_ _ ***F*** _ _ _ ***F*** _ _ _
_ _ _ ***F*** _ _ _ ***F*** _ _
_ _ _ _ ***F*** _ _ _ ***F*** _
_ _ _ _ _ ***F*** _ _ _ ***F***

5 ways to leave 4 spaces between the ***F***s, the ***(EEE)*** can be placed between the ***F***s in 2 ways each:

F _ _ _ _ ***F*** _ _ _ _ → either: ***F (EEE)*** _ ***F*** _ _ _ _ or ***F*** _ ***(EEE) F*** _ _ _ _
_ ***F*** _ _ _ _ ***F*** _ _ _ → either: _ ***F (EEE)*** _ ***F*** _ _ _ or _ ***F*** _ ***(EEE) F*** _ _ _
_ _ ***F*** _ _ _ _ ***F*** _ _ etc…
_ _ _ ***F*** _ _ _ _ ***F*** _
_ _ _ _ ***F*** _ _ _ _ ***F***

4 ways to leave 5 spaces between the ***F***s, the ***(EEE)*** can be placed between the ***F***s in 3 ways each.

3 ways to leave 6 spaces between the ***F***s, the ***(EEE)*** can be placed between the ***F***s in 4 ways each.

2 ways to leave 7 spaces between the ***F***s, the ***(EEE)*** can be placed between the ***F***s in 5 ways each.

1 way to leave 8 spaces between the ***F***s, the ***(EEE)*** can be placed between the ***F***s in 6 ways each.

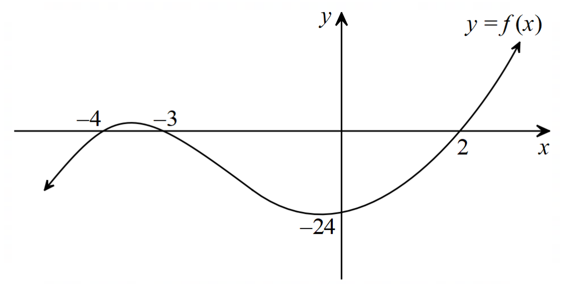
For each of these, 5! ways of arranging the remaining letters:

Total number of ways
$$= (6\times 1+5\times 2+4\times 3+3\times 4+2\times 5+1\times 6)\times 5!$$
$$= 6720$$

Question 13

Sample solution		Suggested marking criteria
(a)	Let $S(n)$ be the statement that $2\times 6\times 10\times\cdots\times(4n-2)=\dfrac{(2n)!}{n!}$ <u>Show $S(1)$ is true:</u> $\text{LHS}=4\times 1-2$ $=2$ $\text{RHS}=\dfrac{(2\times 1)!}{1!}$ $=\dfrac{2!}{1!}$ $=2$ $\therefore S(1)$ is true. <u>Assume $S(k)$ is true, i.e.:</u> $2\times 6\times 10\times\cdots\times(4k-2)=\dfrac{(2k)!}{k!}$ <u>Prove $S(k+1)$ is true, i.e.:</u> $2\times 6\times 10\times\cdots\times(4k-2)\times\left[4(k+1)-2\right]=\dfrac{\left[2(k+1)\right]!}{(k+1)!}$ $2\times 6\times 10\times\cdots\times(4k-2)\times(4k+2)=\dfrac{(2k+2)!}{(k+1)!}$ $\text{LHS}=2\times 6\times 10\times\cdots\times(4k-2)\times(4k+2)$ $=\dfrac{(2k)!}{k!}\times(4k+2)$ $=\dfrac{(2k)!}{k!}\times 2(2k+1)$ $=\dfrac{(2k+1)!}{k!}\times 2$ $=\dfrac{(2k+1)!}{k!}\times\dfrac{2(k+1)}{k+1}$ $=\dfrac{(2k+1)!}{(k+1)!}\times(2k+2)$ $=\dfrac{(2k+2)!}{(k+1)!}$ $=\text{RHS}$ $\therefore S(k+1)$ is true if $S(k)$ is true. Since $S(1)$ was shown true, by the principle of mathematical induction, $S(n)$ is true for all positive integers *n*.	• 3 – correct solution • 2 – uses the inductive hypothesis to attempt to induce the required result • 1 – shows the result is true for the base case

Question 13 (continued)

Sample solution		Suggested marking criteria
(b)	(i) $f(2)=0$ $2^3+(a+2)\times 2^2-2\times 2+b=0$ $8+4(a+2)-4+b=0$ $8+4a+8-4+b=0$ $4a+b+12=0$ $4a+b=-12$	• 1 – correct solution
	(ii) $f(-a)=0$ $(-a)^3+(a+2)\times(-a)^2-2\times(-a)+b=0$ $-a^3+a^2(a+2)+2a+b=0$ $-a^3+a^3+2a^2+2a+b=0$ $2a^2+2a+b=0$	• 1 – correct solution
	(iii) Solving the equations from (i) and (ii) simultaneously: $4a+b=-12 \Rightarrow b=-4a-12$ $2a^2+2a+b=0$ $2a^2+2a-4a-12=0$ $2a^2-2a-12=0$ $a^2-a-6=0$ $(a-3)(a+2)=0$ $a=3 \quad (\text{since } a>0)$ $b=-12-4a$ $=-12-4\times 3$ $=-24$	• 2 – correct solution • 2 – finds the value of a or b
	(iv) $f(x)=x^3+5x^2-2x-24$ $=(x-2)(x^2+7x+12)$ $=(x-2)(x+3)(x+4)$ 	• 2 – correct graph • 1 – correctly factorises $f(x)$

Question 13 (continued)

Sample solution			Suggested marking criteria
(c)	(i)	$y = e^{kx}$ $y' = ke^{kx}$ $y'' = k^2e^{kx}$ If $y = e^{kx}$ is a solutions to $y'' - 2y' - 3y = 0$, then: $y'' - 2y' - 3y = 0$ $k^2e^{kx} - 2ke^{kx} - 3e^{kx} = 0$ $e^{kx}\left(k^2 - 2k - 3\right) = 0$ $k^2 - 2k - 3 = 0 \quad \left(\text{since } e^{kx} \neq 0\right)$	• 2 – correct solution • 1 – correctly finds y' and y'' for $y = e^{kx}$
	(ii)	$k^2 - 2k - 3 = 0$ $(k-3)(k+1) = 0$ $k = -1, k = 3$ Therefore, solutions to $y'' - 2y' - 3y = 0$ of the form $y = e^{kx}$ are $y = e^{-x}$ and $y = e^{3x}$.	• 2 – correct solution • 1 – correctly solves $k^2 - 2k - 3 = 0$ – solves $k^2 - 2k - 3 = 0$ to determine solutions to the differential equation of the form $y = e^{kx}$
	(iii)	Suppose $k_1 = -1, k_2 = 3$ (interchangeable without affecting the result). We need to show that $y = Ae^{-x} + Be^{3x}$ is also a solution to $y'' - 2y' - 3y = 0$. $y = Ae^{-x} + Be^{3x}$ $y' = -Ae^{-x} + 3Be^{3x}$ $y'' = Ae^{-x} + 9Be^{3x}$ $y'' - 2y' - 3y = Ae^{-x} + 9Be^{3x} - 2\left(-Ae^{-x} + 3Be^{3x}\right) - 3\left(Ae^{-x} + Be^{3x}\right)$ $= Ae^{-x} + 2Ae^{-x} - 3Ae^{-x} + 9Be^{3x} - 6Be^{3x} - 3Be^{3x}$ $= 0$ Therefore, $y = Ae^{-x} + Be^{3x}$ is also a solution to $y'' - 2y' - 3y = 0$. Alternatively, without recognising the substitution $k_1 = -1, k_2 = 3$: $y = Ae^{k_1x} + Be^{k_2x}$ $y' = Ak_1e^{k_1x} + Bk_2e^{k_2x}$ $y'' = A\left(k_1\right)^2 e^{k_1x} + B\left(k_2\right)^2 e^{k_2x}$ $y'' - 2y' - 3y$ $= A\left(k_1\right)^2 e^{k_1x} + B\left(k_2\right)^2 e^{k_2x} - 2\left(Ak_1e^{k_1x} + Bk_2e^{k_2x}\right) - 3\left(Ae^{k_1x} + Be^{k_2x}\right)$ $= A\left(k_1\right)^2 e^{k_1x} - 2Ak_1e^{k_1x} - 3Ae^{k_1x} + B\left(k_2\right)^2 e^{k_2x} - 2Bk_2e^{k_2x} - 3Be^{k_2x}$ $= Ae^{k_1x}\left[\left(k_1\right)^2 - 2k_1 - 3\right] + Be^{k_2x}\left[\left(k_2\right)^2 - 2k_2 - 3\right]$ $= 0$ since k_1 and k_2 are solutions to $k^2 - 2k - 3 = 0$ by definition Therefore, $y = Ae^{k_1x} + Be^{k_2x}$ is also a solution to $y'' - 2y' - 3y = 0$.	• 2 – correct solution • 1 – correctly finds y' and y'' for $y = Ae^{-x} + Be^{3x}$

Question 14

	Sample solution	Suggested marking criteria
(a)	$\dfrac{\lvert \underset{\sim}{p}+\underset{\sim}{q}\rvert^2+\lvert \underset{\sim}{p}-\underset{\sim}{q}\rvert^2}{\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2}=\dfrac{(\underset{\sim}{p}+\underset{\sim}{q})\cdot(\underset{\sim}{p}+\underset{\sim}{q})+(\underset{\sim}{p}-\underset{\sim}{q})\cdot(\underset{\sim}{p}-\underset{\sim}{q})}{\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2}$ $=\dfrac{\underset{\sim}{p}\cdot\underset{\sim}{p}+\cancel{\underset{\sim}{p}\cdot\underset{\sim}{q}}+\cancel{\underset{\sim}{q}\cdot\underset{\sim}{p}}+\underset{\sim}{q}\cdot\underset{\sim}{q}+\underset{\sim}{p}\cdot\underset{\sim}{p}-\cancel{\underset{\sim}{p}\cdot\underset{\sim}{q}}-\cancel{\underset{\sim}{q}\cdot\underset{\sim}{p}}+\underset{\sim}{q}\cdot\underset{\sim}{q}}{\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2}$ $=\dfrac{\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2+\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2}{\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2}$ $=\dfrac{2\left(\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2\right)}{\lvert \underset{\sim}{p}\rvert^2+\lvert \underset{\sim}{q}\rvert^2}$ $=2$ *Note:* $\lvert \underset{\sim}{p}\pm\underset{\sim}{q}\rvert^2\neq\lvert \underset{\sim}{p}\rvert^2\pm2\lvert \underset{\sim}{p}\rvert\lvert \underset{\sim}{q}\rvert+\lvert \underset{\sim}{q}\rvert^2$. *Deduct 1 mark if students utilised this.*	• 2 – correct solution • 1 – correctly demonstrates the use of the identity $\lvert \underset{\sim}{u}\rvert^2=\underset{\sim}{u}\cdot\underset{\sim}{u}$
(b)	(i) $u=9-\sqrt{h}$ $\sqrt{h}=9-u$ $h=(9-u)^2$ $\dfrac{dh}{du}=-2(9-u)$ $\displaystyle\int\frac{dh}{9-\sqrt{h}}=\int\frac{-2(9-u)}{u}\,du$ $=2\displaystyle\int 1-\frac{9}{u}\,du$ $=2\left(u-9\ln\lvert u\rvert\right)+c_1$ $=2u-18\ln\lvert u\rvert+c_1$ $=2\left(9-\sqrt{h}\right)-18\ln\left\lvert 9-\sqrt{h}\right\rvert+c_1$, for some constant c_1 $=18-2\sqrt{h}-18\ln\left\lvert 9-\sqrt{h}\right\rvert+c_1$ $=-18\ln\left\lvert 9-\sqrt{h}\right\rvert-2\sqrt{h}+c$, where $c=c_1+18$ is a constant	• 2 – correct solution • 1 – uses the given substitution to express the integrand in terms of u
	(ii) Since these are growing tress, $\dfrac{dh}{dt}>0$ $\dfrac{t^{0.5}\left(9-\sqrt{h}\right)}{18}>0$ $9-\sqrt{h}>0$ $\sqrt{h}<9$ $h<81$ From $9-\sqrt{h}$, $h\geq0$. $\therefore 0\leq h<81$	• 1 – correct answer

Question 14 (continued)

Sample solution		Suggested marking criteria
(b)	(iii) $$\frac{dh}{dt}=\frac{t^{0.5}\left(9-\sqrt{h}\right)}{18}$$ $$\int_1^{16}\frac{dh}{9-\sqrt{h}}=\frac{1}{18}\int_0^t t^{0.5}\,dt$$ $$\left[-18\ln\left\|9-\sqrt{h}\right\|-2\sqrt{h}\right]_1^{16}=\frac{1}{18}\left[\frac{t^{1.5}}{1.5}\right]_0^t$$ $$\left(-18\ln\left\|9-\sqrt{16}\right\|-2\sqrt{16}\right)-\left(-18\ln\|8\|-2\right)=\frac{t^{1.5}}{27}$$ $$-18\ln 5-8+18\ln 8+2=\frac{t^{1.5}}{27}$$ $$27\left(18\ln\frac{8}{5}-6\right)=t^{1.5}$$ $$t=\left[27\left(18\ln\frac{8}{5}-6\right)\right]^{\frac{2}{3}}$$ $$=16.4\text{ years }\left(3\text{ sig. fig.}\right)$$	• 3 – correct solution • 2 – correctly integrates and substitutes the given initial values into the primitives • 1 – correctly separates the variables of the differential equation (constants can be on either side)
(c)	(i) Let X denote the binomial random variable that counts the number of rose bushes that produce pink flowers with parameters $n=10$, $p=0.15$, where p is the probability of a particular rose bush sprouting pink roses. $P(X=3)={}^{10}C_3\times 0.15^3\times 0.85^7$ $=0.129833...$ $=13\%$ (nearest per cent)	• 1 – correct solution
	(ii) In n trials, $P(X\geq 1)>0.95$ $1-P(X=0)>0.95$ $P(X=0)<0.05$ ${}^{n}C_0\times 0.15^0\times 0.85^n<0.05$ $0.85^n<0.05$ $n\ln 0.86<\ln 0.05$ $n>\frac{\ln 0.05}{\ln 0.86}$ $n>18.4331...$ $n=19$ Therefore, at least 19 rose bushes need to be bought.	• 2 – correct solution • 1 – establishes a suitable inequation involving binomial probability

Question 14 (continued)

Sample solution			Suggested marking criteria
(c)	(iii)	For $X \sim \text{Bin}(125, 0.15)$: $\mu = np$ $= 125 \times 0.15$ $= 18.75$ $\sigma^2 = npq$ $= 125 \times 0.15 \times 0.85$ $= 15.9375$ The binomial distribution of X can be approximated by the normal distribution Y where $Y \sim N(18.75\ ,\ 15.9375)$. $P(20 < X \leq 30) = P(20 \leq X \leq 30)$ $\approx P(20 \leq Y \leq 30)$ (without continuity correction) $= P(Y \leq 30) - P(Y \leq 20)$ $= P\left(Z \leq \frac{30 - 18.75}{\sqrt{15.9375}}\right) - P\left(Z \leq \frac{20 - 18.75}{\sqrt{15.9375}}\right)$ $\approx P(Z \leq 2.82) - P(Z \leq 0.31)$ $\approx 0.9976 - 0.6217$ $= 0.3759$ $= 37.6\%$ (1 d.p.) $P(20 < X \leq 30) = P(20 \leq X \leq 30)$ $\approx P(19.5 \leq Y \leq 30.5)$ (with continuity correction) $= P(Y \leq 30.5) - P(Y \leq 19.5)$ $= P\left(Z \leq \frac{30.5 - 18.75}{\sqrt{15.9375}}\right) - P\left(Z \leq \frac{19.5 - 18.75}{\sqrt{15.9375}}\right)$ $\approx P(Z \leq 2.94) - P(Z \leq 0.19)$ $\approx 0.9984 - 0.5753$ $= 0.4231$ $= 42.3\%$ (1 d.p.)	• 4 – correct solution • 3 – correctly applies *z*-score formulas • 2 – evaluates mean and variance of the binomial distribution • 1 – evaluates mean or variance of the binomial distribution